Excitation of Alfvén modes by Energetic Particles in Magnetic Fusion

presented by N.N. Gorelenkov

PPPL, Princeton University

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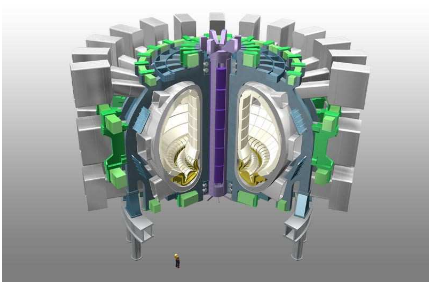
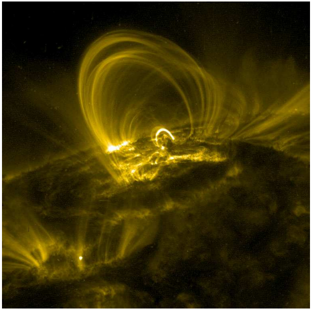
Cadarache, France
ITER, June 20, 2011

Fusion is vital for mankind to survive

controlled fusion on earth (ITER) vs. therm. fusion in the sun

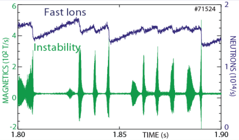


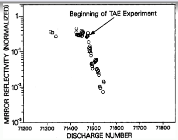


Motivations to study energetic particle (EP) physics

- Heating for Deuterium-Tritium (DT) reactor: 1 GW DT plasma should have 200MW (20%) in fusion alpha particles.
- *Where does this power go ... e,i, lost?*
- *What are the mechanisms of power transfer?*
- *How to control it? Can we design an efficient fusion reactor?*
- *Can it damage the plasma facing components, diagnostics?*
- *Can we make use of EP instabilities for plasma diagnostics?...*

  Lecture is far from systematic review, some names/labs are omitted, not consistent...





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Carbon coats DIII-D mirrors when escaping fast ions ablate the graphite wall. **Need to mitigate the risk: ITER cost is >$10bln.**

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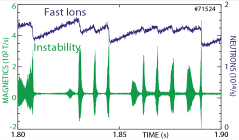

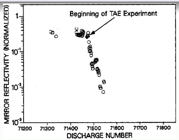

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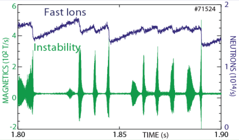

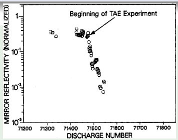

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1. Motivations
   - "thermonuclear" instabilities
2. From single particle to collective effects
   - sources, parameters, distr. function...
   - global MHD codes are brought for studies
3. Alfvén (+other) waves, eigenmodes
   - Local analysis
   - Continuum gaps
   - Mode structure
   - Lower frequency BAAE gaps
4. Fast ion drive of Alfvén modes
   - n of unstable mode scales with ω_*, R
   - experimental confirmation of growth scalings
   - apply scalings to ITER plasma

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Typical EP sources in present day (PD) experiments

Sources of energetic particles (ions):

- resonant frequency heating (RF): H, $50 \div 1000 keV$,
- beam injection (NBI): D, H, $50 - 350 keV$ ($1 MeV$ in ITER),
- fusion products: $\alpha(3.52 MeV, DT)$, $p(3 MeV)$, $T(1 MeV, DD)$...
- test particles,

EP, super-thermal ions: $E_{EP} \gg T_{pl}$, $v_{Ti} < v_{EP} \ll v_{Te}$
Relation to Alfvén velocity, $v_A = B/\sqrt{4\pi\rho_i}$, *in fusion plasmas*
$v_{Ti} = \sqrt{\beta_i} v_A < v_A \leq v_{EP}$

Corollary

- *EPs are decoupled from background ions for* $E_{EP} > 20 T_e$, *typically only electron drag (or RF) is present.*
- *characteristic drift, transit frequencies are much larger than those of thermal ions.*

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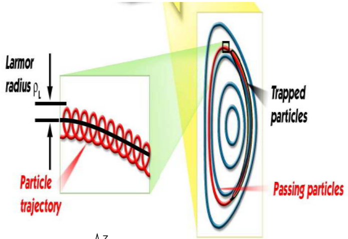
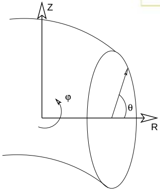
How EPs are confined? What is special?

Schematic of charged particle orbits in a tokamak $\rho_{Li} \ll \rho_{Lf} \ll R$





Plasma current estimate

- Larmor radius should be small enough (1) $\rho_{Lf} \ll a$, $\rho_{Lf} = v/\omega_c = vmc/eB_\varphi$
- Poloidal Larmor radius (drift orbit width) determines confinement $\Rightarrow$ (2) $a > \rho_{Lf\theta} = v_{\parallel} mc/eB_\theta$.
- Find required plasma current via: $B_\theta\,[T] = \left(\frac{1}{2}\right) 0.2 I_{pl}[A]/a[cm]$ adiabatic moment conservation $v_\perp^2/B \simeq const \Rightarrow v_{\parallel} \simeq v\sqrt{2a/R}\ \ / \Rightarrow$ $I_{pl}\mathrm{crit}\ \ [A] = \frac{10\rho_{L\alpha}[cm]}{\sqrt{R/2a}} B\,[Gauss] \simeq 2\times 10^6$
- In ITER plasma $I_{pl} = 10MA \gg I_{pl crit}$.
- *Ramp up regime can violate this.*

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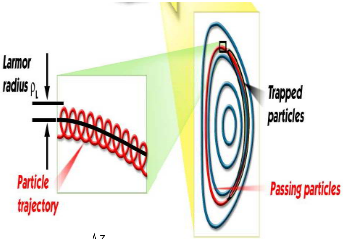

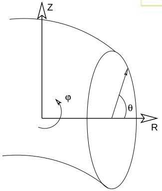

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What is classical steady state EP distribution function

For drift-kinetic stationary equation, $\partial f / \partial t = 0$

$$\frac{1}{\tau_{se} v^2} \frac{\partial}{\partial v} \left(v^3 + v_*^3 \right) f + S(v) = 0$$

If $S(v) = (4\pi)^{-1} S_0 \delta(v - v_0)$, such as alpha's $S_0 = \langle \sigma v \rangle n_D n_T$:

$$\frac{1}{v^2} \frac{\partial}{\partial v} \left(v^3 + v_*^3 \right) f + \frac{\delta(v - v_0)}{4\pi} S_0 \tau_{se} = 0,$$

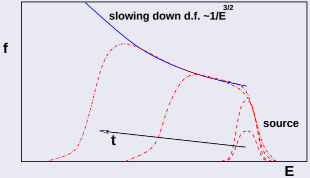
has a solution

$$f = \frac{S_0 \tau_{se}}{4\pi} \frac{v_0^2 H(v_0 - v)}{v^3 + v_*^3}.$$

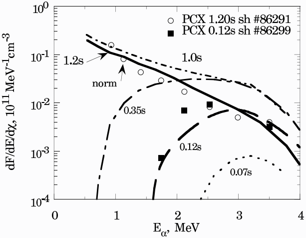
H is $\sim$ step function.

Classical EP confinement is demonstrated in DT TFTR plasma

if plasma is not in steady state, i.e. beam like
(fast start of the discharge)



TFTR *fusion* alphas



Numerical simulations of NPA in TFTR: more effects are included: (1) realistic geometry, (2) $S = S(v,t)$ *(Gorelenkov, NF'97)*

Is it stable?

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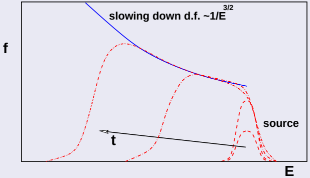

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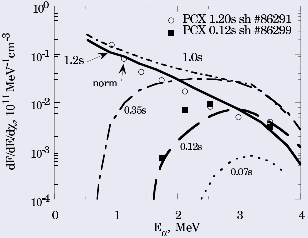

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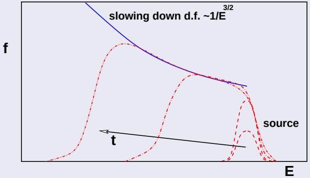

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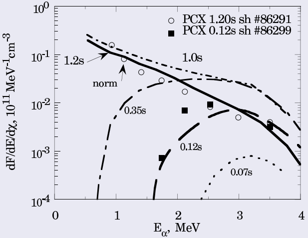

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Outline

"New" development came from powerfull MHD codes

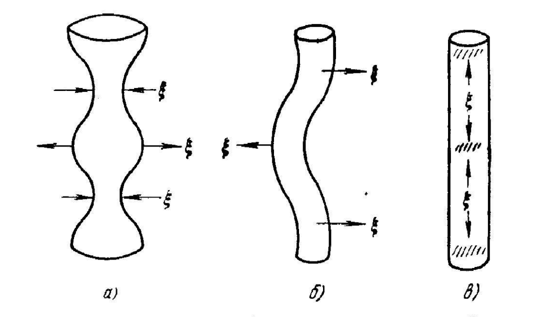
such as NOVA, CASTOR > 1986 ...
good place, good time ...
MHD was well applicable for such "novel" problems:

- established codes, PEST, GATO, KINX, ...
- benchmarked against analytical solutions
- ——//—— other codes
- good problem for young researchers

Outline

Three types of plasma oscillations can be of interest

compr. Alfvén shear Alfvén sound waves



Charactertic wave velocities: $\omega = k_\parallel v_A$ or $\omega = k_\parallel v_s$

Shear Alfvén equation in nonuniform plasma

Use local Alfvén dispersion $\omega^2/v_A^2 - k_\parallel^2 = 0$.
Operate with

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

on equation for electrostatic potential or $(div)(grad)$; $\left(\frac{\omega^2}{v_A^2} - k_\parallel^2\right)\phi = 0$.

We find shear Alfvén equation (AE)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\omega^2}{v_A^2} - k_\parallel^2\right) r \frac{\partial}{\partial r}\phi + \left(\frac{\omega^2}{v_A^2} - k_\parallel^2\right)\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\phi + Q\phi = 0.$$

Some other terms are included in Q, such as ballooning term.
Each radial point of $\omega_A^2(r) = v_A^2 k_\parallel^2(r)$ corresponds to (AE) singularity and represents the continuum via v_A and $k_\parallel$.

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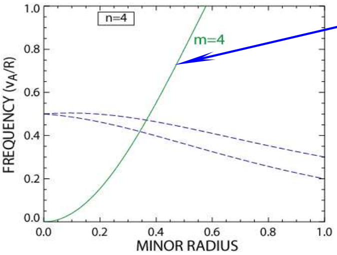

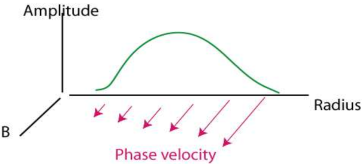

(Heidbrink, '08)

local Alfvén continuum $\omega_A(r)$.

Radially localized kinetic modes (KAW) are strongly damped

$$\gamma \sim d\left(k_{\parallel} v_A\right)/dr$$

Mechanism of the damping is through strong parallel electric field $\Leftarrow$ thermal ion FLR + small scale.

No much interest!

Rosenbluth'75, Mikhailovskii'75

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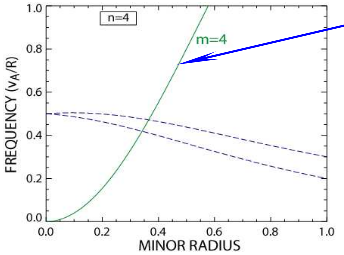

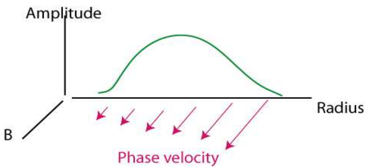

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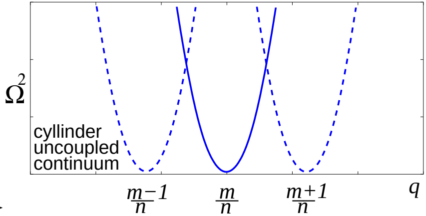
Shear Alfvén continuum schematic (cylinder, zero β)

Magnetic field variation shapes the continuum through the safety factor profile $q = B_\varphi r / B_\theta R$,

$$\hat{k}_\parallel \phi = -i \frac{\vec{B}}{B} \cdot \vec{\nabla} \sum_m \phi_m e^{-i\omega t + im\theta - in\varphi} = \sum_m \left(\frac{m}{q(r) R} - \frac{n}{R} \right) \phi_m e^{\cdots}$$



$B = B(\theta, r)$, $\omega_A^2(r) = v_A^2 k_\parallel^2 \Rightarrow$
Rational surfaces correspond to $k_{\parallel m} = 0$.
Solutions should account for the continuum structure.
If harmonics are coupled (nonhomogeneous) continuum gaps appear.

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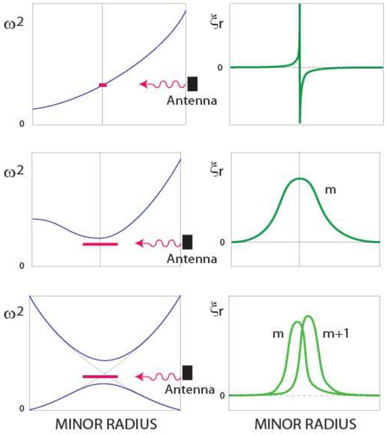

Pinches, PhD Thesis.

- antenna, *local AE are highly damped*
- r-extended solution below local extremum (cylinder)
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- Gap creates conditions for broad mode

Natural instability can exist if damping is not high: (i) gap; (ii) near extremum (STs);

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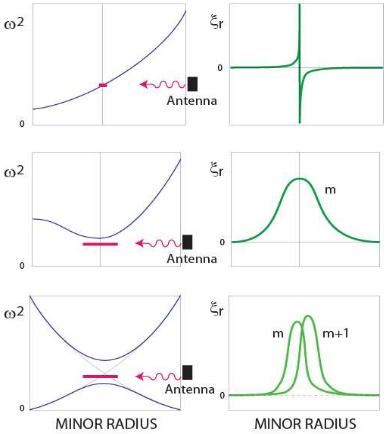

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Why continuum gaps are important?

Gaps in the continua are common phenomena.
Example: energy spectrum gaps of valence electrons in a periodic potential well of a crystal lattice.

- Higher order effects are tested as coupling is described by small parameters: toroidicity $\frac{a}{R}$; pressure β:... great test for theories,
- global modes may exist with frequencies inside gaps
- if modes with frequencies in the gaps exist they have low damping: no continuum damping from kinetic, singular mode.
- Can help to organize Alpha channeling (r-diffusion) together with the RF antennae (E-diffusion - N. Fisch talk)

Realistic shear Alfvén continuum (schematic, torus)

Again take the eigenmode equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\omega^2}{v_A^2}-k_\parallel^2\right)r\frac{\partial}{\partial r}\phi+\left(\frac{\omega^2}{v_A^2}-k_\parallel^2\right)\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\phi+Q\phi=0.$$

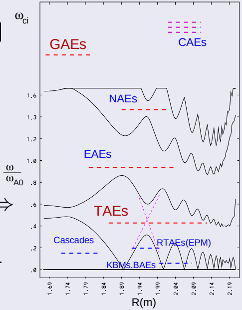
If pol-/toroid. harmonic expansion is applied

$$\phi=\sum_m \phi_m exp\left[-i\omega t+im\theta-in\varphi\right],$$

geometrical effects couple harmonics:

$$v_A^2k_\parallel^2=B^2\left(r,\theta\right)k_\parallel^2/4\pi\rho;B=B_0\left(1+\varepsilon cos\theta...\right)\Rightarrow$$

(high order) toroidicity, ellipticity, triangularity, ..., pressure induced gaps.



(Cheng,Chance'86,K.L.Wong'91,W.Heidbrink'91)

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General procedure for ideal MHD codes

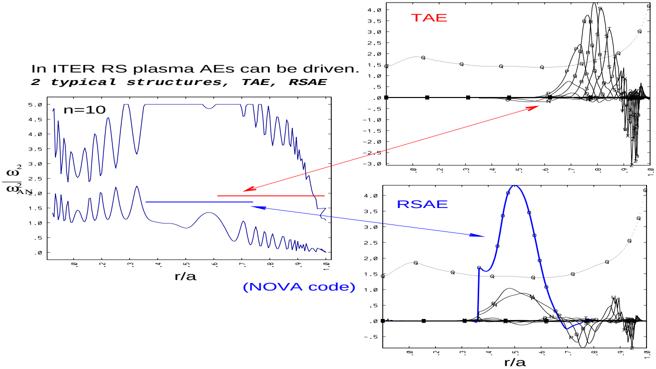
Based on m-truncated expansion
$\phi = \sum_{m,l}^{M,L} \phi_{m,l} U_l(\psi) exp[-i\omega t + im\theta - in\varphi]$,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\omega^2}{v_A^2} - k_\parallel^2\right) r \frac{\partial}{\partial r} \begin{pmatrix} \cdots \\ \phi_m \\ \phi_{m+1} \\ \cdots \end{pmatrix} + \left(\frac{\omega^2}{v_A^2} - k_\parallel^2\right) \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \begin{pmatrix} \cdots \\ \phi_m \\ \phi_{m+1} \\ \cdots \end{pmatrix} +$$

$$+Q \begin{pmatrix} \cdots \\ \phi_m \\ \phi_{m+1} \\ \cdots \end{pmatrix} = 0 \quad \Rightarrow$$

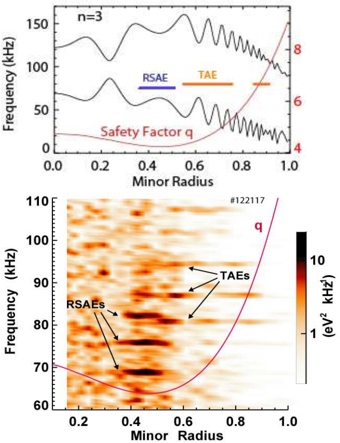
Sophisticated geometrical effects are present.
Non MHD effects can be present perturbatively on nonperturbatively: FLR, kinetic ...
Analytic theories employ sometimes similar approach.

Generally TAE structure is complicated in realistic geometry



RSAEs and TAEs can be unstable, lead to α, beam losses in ITER. Both modes are known to induce beam losses in DIII-D.

A remarkable agreement between TAE, RSAE mode structure in DIII-D and NOVA modeling (ideal MHD)



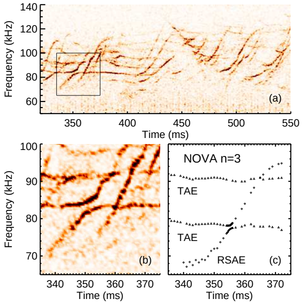
- NBI into ∼low density DIII-D plasma
- Various gap modes reside in an effective waveguides, such as caused by q_{min} presence
- RSAEs are localized near continuum extrema points
  - RSAEs have one dominant poloidal harmonic
- Both RSAE and TAE structures agree quantitatively with ideal MHD (NOVA simulations)

Van Zeeland'06 PRL, '07 PhysPlasmas.

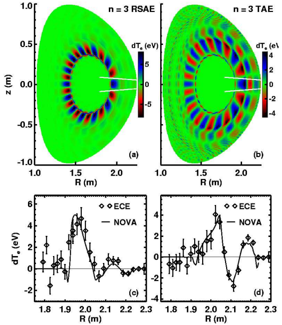
Ideal MHD TAE structure agrees with ECE

Internal TAE/RSAEs mode structures measured by ECE show excellent agreement with ideal MHD predictions *(NOVA calculations, Van Zeeland, PRL '06)*

RSAE & TAE frequencies crossover



RSAE & TAE structures



It could be that earlier works *(Carolipio'01, Heidbrink'97)* dealt with other modes such as EPMs in high fast ion beta plasma.

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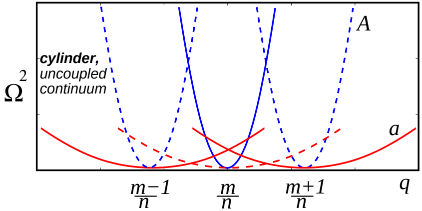
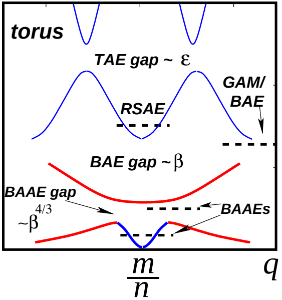
Plasma pressure introduces BAAE (Alfvén-acoustic) gaps/modes

- Alfvén (A) continuum at low frequency: $\Omega^2 = v_A^2 k_{0,\pm1}^2 \, / \left(1+2q^2\right)$ (modified)
- Acoustic (a) branch $\Omega^2 = \left[\gamma\beta v_A^2/2\left(1+\delta\right)\right] k_{0,\pm1}^2$ is coupled via $m\pm1$ sidebands with modified Alfvén continuum (m harmonic) due to geodesic curvature and pressure.



$\Rightarrow$gaps



BAAE gaps are due to β and geodesic curvature effects

Global modes exist in A-a gaps *Chu'92, Van der Holst'01, Gorelenkov'07*

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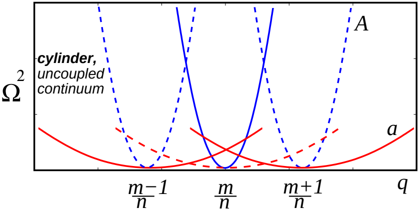

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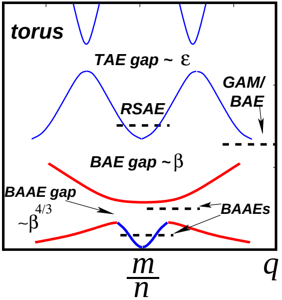

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Alfvén/acoustic continuum bounds global modes

Shear Alfvén and acoustic continua equations capture main effects in low-β, large aspect ratio plasma, low ω_*, *(Cheng, Chance, PFl '86):*

$$\Omega^2 y + \partial_\parallel^2 y \quad + \gamma\beta \sin\theta\, z \quad = 0\,(Alfvenic) \qquad (1)$$

$$\Omega^2 \left(1 + \frac{\gamma\beta}{2}\right) z + \frac{\gamma\beta}{2}\partial_\parallel^2 z \quad + 2\Omega^2 \sin\theta\, y \quad = 0\,(acoustic), \qquad (2)$$

where $\Omega \equiv \omega R / v_A$, $y \equiv \xi_s \varepsilon / q$, $\xi_s \equiv \vec{\xi} \cdot \frac{[\mathbf{B}\times\nabla\psi]}{|\nabla\psi|^2}$ and $z \equiv \nabla \cdot \vec{\xi}$, $\hat{k}_\parallel \equiv i\partial_\parallel / R$.

Geodesic curvature coupling: m Alfvénic and $m \pm 1$ acoustic harmonics.

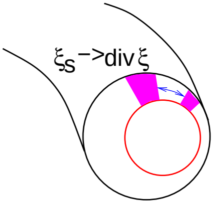
Various solutions exist

uncoupled ***acoustic*** **(a)** $\Omega^2 = \frac{1}{2}\gamma\beta k_\parallel^2 R^2$

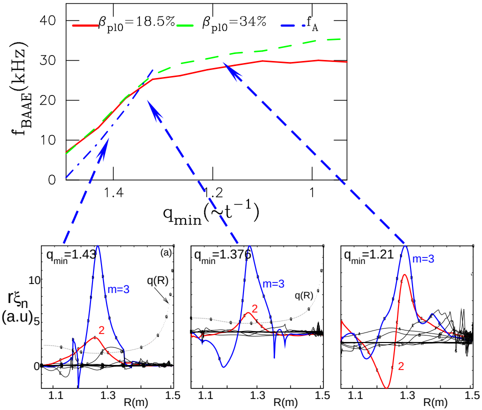
and ***Alfvénic*** **(A)** branches $\Omega^2 = k_\parallel^2 R^2 + \Omega_{GAM}^2$.

***GAM*s**: $\Omega_{GAM}^2 = \gamma\beta\left(1 + 1/2q^2\right)$

modified shear Alfvén branch $\Omega^2 = \frac{k_\parallel^2 R^2}{1+2q^2}$



NOVA: gap BAAE broadens as q_{min} decreases in NSTX

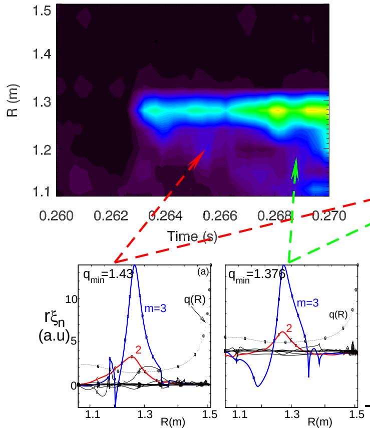


- BAAE frequency sweeps as q-profile relaxes.
  - f does not depend on beta near rational q_{min} =1.5.
  - ξ_r has one dominant harmonic $m = nq_{min} = 3$.
- f_{BAAE} is close to modified Alfvén branch $f_A = v_A k_{\|}/2\pi\sqrt{1+2q^2_{min}}$
- Continuously transforms to broad gap mod
- Gaps shape BAAEs

Ultra SXR measures the same radial structure broadening

Raw USXR signal (∼BAAE structure, *Tritz, JHU*)

Radial profile evolution



BAAE broadens as q_{min} decreases

Other tokamaks observed BAAEs

- JET, DIII-D, ASDEX ...

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EP driven instabilities

Three conditions should be satisfied:

- source of free energy must be present
- particles should be at resonance
- drive should be *higher than the damping*

Free energy sources:

- pressure radial gradient (universal drive), low frequency
- velocity space gradient, high frequency
    - inversed velocity distribution
    - velocity anisotropy

As an example we construct the universal instability drive

$$\frac{\gamma}{\omega} = K^{-1}\left[\Delta\mathscr{E}\beta_\alpha\frac{\partial f\left(v_{res}\right)}{\partial\mathscr{E}_\alpha} + \frac{\Delta r}{r}\frac{\partial\beta_\alpha}{\partial\ln r}f\left(v_{res}\right)\right] \sim -\omega\beta_\alpha f\left(v_{res}\right) + \frac{n}{\omega_c}\frac{\partial\beta_\alpha}{\partial\ln r}f\left(v_{res}\right) \sim n.$$

Slow ramp up of the alpha source ⇒ avoid thermonucl.instability

Most unstable mode number scales with the machine size

Among possible Alfvén eigenmodes toroidicity-induced AE (TAEs) modes are likely to limit fusion product confinement in BP.

Low frequency modes are mostly responsible for fast ion radial transport: TAEs, EPMs, BAEs.

High frequency modes are responsible for phase space particle diffusion: GAEs, CAEs, ICE.

Theory:

n range of most unstable TAEs determined by Finite Orbit Width (FOW) effects: (Berk PL,'92, Fu PF, '92, Breizman'95, Candy'95)

$$k_\perp \Delta_b \simeq \frac{nq^2\rho_b}{r} \sim 1 \,|\Rightarrow \frac{r}{R} n_{max} < n < n_{max} \simeq \frac{r\omega_{c\alpha}}{q^2 v_A}$$

Codes (NOVA) show agreement with theory

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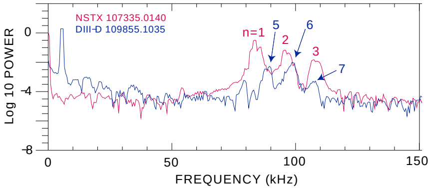
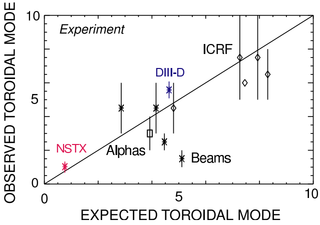
Experimental observations confirm theory predictions

DIII-D/NSTX similarity experiments were designed to confirm theory predictions *(W. Heidbrink, PPCF '03)*:

- The $a = 0.8m$ radius in NSTX and DIII-D but different $R = 1m$ vs $R = 1.7m$, $B = 0.6T$.
- Use similar NBI features: injection geometry, energy, trapped to passing particle ratio.

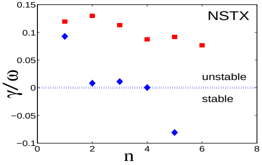


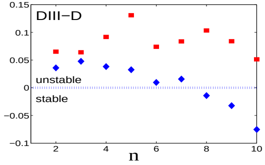


Most unstable mode number scales as

$$n \sim a/q^2$$

Trend of unstable TAE's n-dependence is recovered by theory



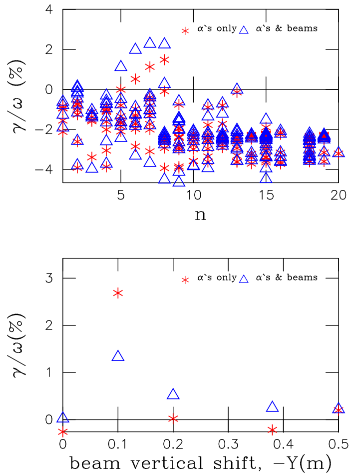


■ no damping, ♦ - with damping

- TAEs excitation thresholds are reproduced for medium-n numbers
- NOVA-K with isotropic d.f. does not predict observed unstable modes
- Main damping mechanisms are:
    - ion Landau damping (dominant in ITER), radiative damping
- Stabilization of TAEs at high end of n range is due to FOW effects and higher damping
- **Helps to validate predictive capabilities of theory.**

Outline

Numerical simulations of AE stability in ITER

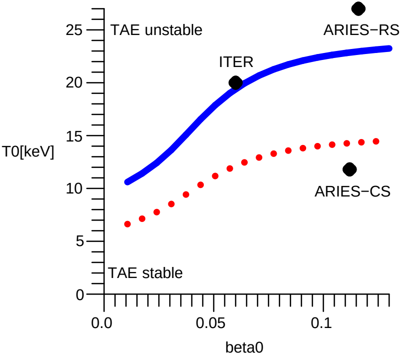


1 MeV beam ion drive is comparable with the $\alpha-$drive.

- lowering NBI energy to 0.5 MeV significantly reduces drive.
- medium, high-n's are unstable $n = 6 - 13$.
- most unstable modes are localized at $r/a \sim 0.5$ and avoid central ion Landau damping.
- TAE control is possible via NBI aiming.

Projections to ITER and other BPs designs

Analytical model for TAE stability in BPs with drive from beams and alphas



- Stability diagram in $\beta_{pl}(0)/T_i(0)$
  - source of alphas comes from background plasma $S_\alpha \sim \langle\sigma v\rangle \Rightarrow \frac{-\partial\beta_\alpha}{\partial \ln r} \simeq \frac{7}{2}\beta_\alpha \frac{-\partial \ln T}{\partial \ln r}$
  - theory (dashed curve), normalized to NOVA (solid curve)
- α's slowing down d.f., ion Landau and trapped electron collisional dampings.
- *see K.Ghantous poster for details.*

The Status

Linear theory

- well developed, routinely finds zoo of modes
- theory is validated
- problems exist: kinetic effects, damping mechanisms are not tested, comprehensive codes are buit.

Non-linear theory

- Nonlinear/transport with multiple instabilities should be #1 priority:
  - Develop realistic reduced models
  - Realistic, global nonlinear codes
  - V&V needed (ITPA?)
- Experiments on the "sea of Alfven modes" driven transport should be done on present day machines.

Effect of instabilities on energetic ions depends on frequency

If cyclotron interaction is ignored we have

$$\frac{\Delta P_\varphi}{n_\varphi} = \frac{\Delta \mathcal{E}_{EP}}{\omega}$$

or

$$\frac{\Delta r}{r}\frac{\mathcal{E}_{EP}}{\Delta \mathcal{E}_{EP}} = \left(\frac{nq}{r}\rho_{EP}\right)\left(\frac{\rho_{EP}}{2r}\frac{\omega_c}{\omega}\right).$$

- Low frequency instabilities ($< 200kHz$) may induce radial particle transport
- High frequency instabilities affect velocity space transport.